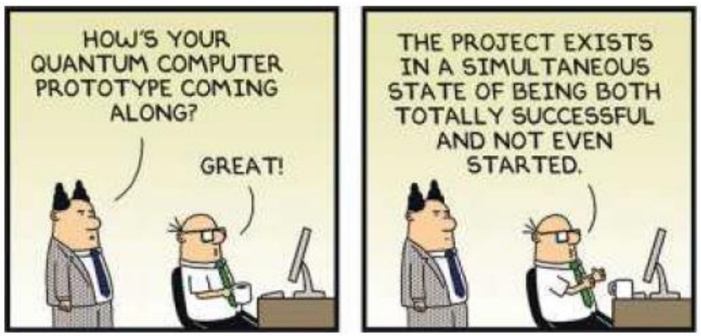
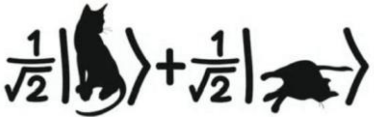


Scott Adams/Dilbert

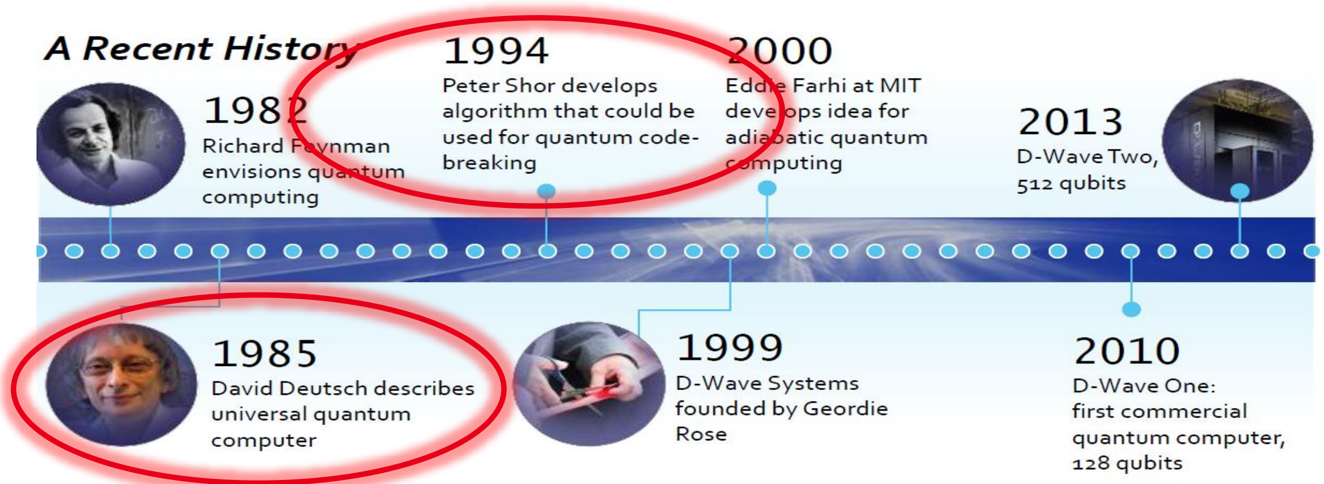
VERT Daniel
LOUISE Stephane
SIRDEY Renaud

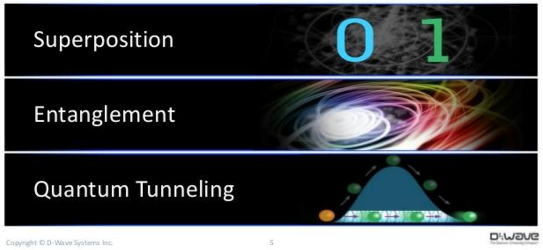
ON THE LIMITATIONS OF THE CHIMERA GRAPH TOPOLOGY IN USING ANALOG QUANTUM COMPUTERS



BACKGROUND AND MOTIVATIONS

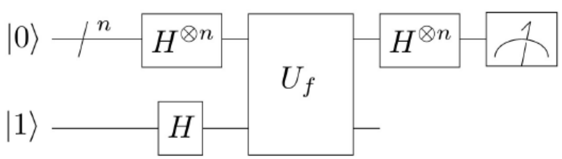
« I think I can safely say that nobody understands quantum mechanics » ***Richard Feynman***





Deutsch-Jozsa (1992)

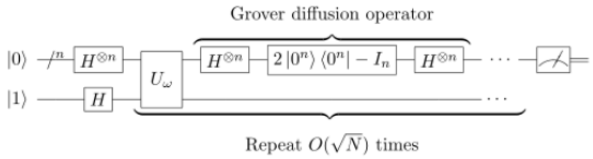
- Guess the oracle



Check if f: {0.1}^{n} → {0.1} is symmetric using a quantum computer

Grover (1996)

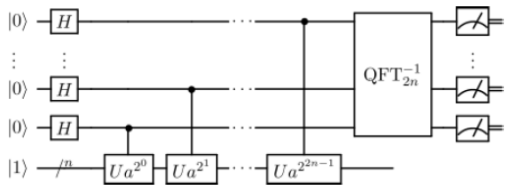
- Search



Search in an unsorted database with N time entries $O(N^{1/2})$

Shor (1994)

- Inverse Log - Factorization



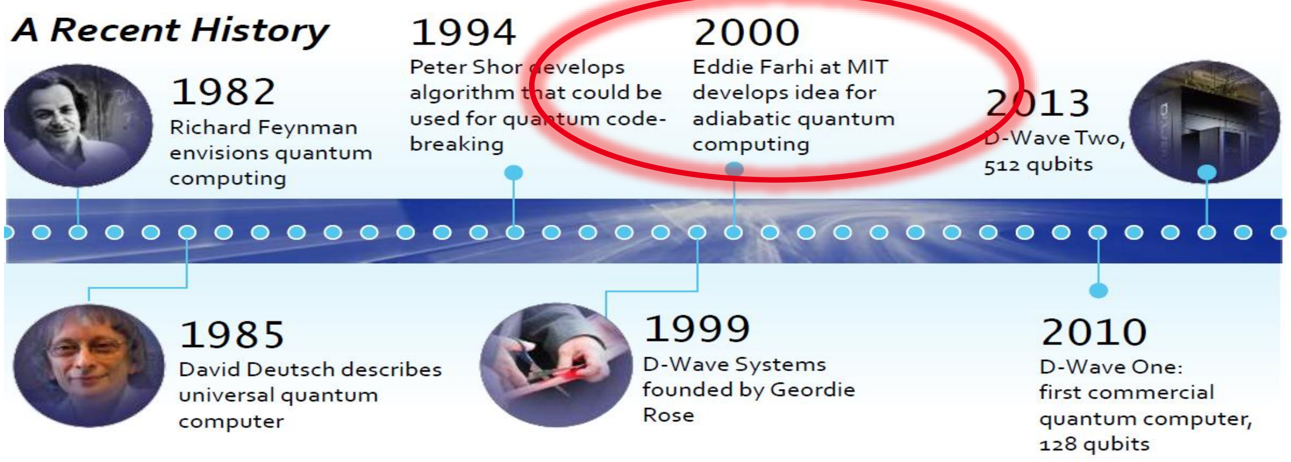
For an integer N, find its prime factors

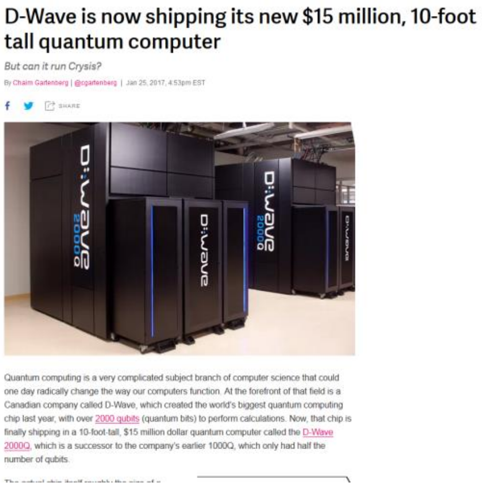
LIMITATION: Remain very modest in terms of informatics*.

No clear technological path for scaling up, maybe in very long term.

* "49" qbits at IBM

BACKGROUND AND MOTIVATIONS




D-Wave is now shipping its new $15 million, 10-foot tall quantum computer

But can it run Crysis?

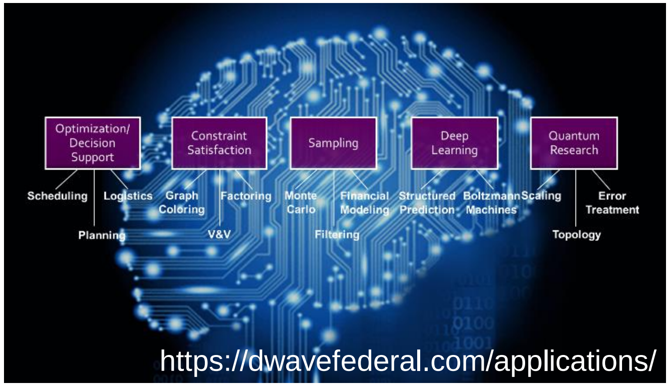
By Chaim Gartenberg | @cgartenberg | Jan 25, 2017, 4:53pm EST

SHARE

Quantum computing is a very complicated subject branch of computer science that could one day radically change the way our computers function. At the forefront of that field is a Canadian company called D-Wave, which created the world's biggest quantum computing chip last year, with over 2000 qubits (quantum bits) to perform calculations. Now, that chip is finally shipping in a 10-foot-tall, $15 million dollar quantum computer called the D-Wave 2000Q, which is a successor to the company's earlier 1000Q, which only had half the number of qubits.

<u>APPLICATIONS:</u> Naturally applies to the optimization / operational research problems

<u>LIMITATION</u>: Minimize an Hamiltonian (spin glasses)* using a quantum phenomenon, functionally similar to simulated annealing

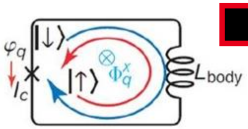


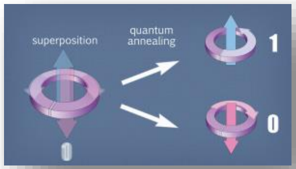
D-Wave Two	D-Wave 2X	D-Wave 2000Q
512 (8x8x8) qubit "Vesuvius" processor	**1152** (8x12x12) qubit "Washington" processor	**2048** (8x16x16) qubit "Whistler" processor
509 qubits working – 95% yield	**1097** qubits working – 95% yield	**2038** qubits working – 97% yield
1472 *J* programmable couplers	**3360** *J* programmable couplers	**6016** *J* programmable couplers
20 mK max operating temperature (**18 mK** nominal)	**15 mK** max operating temperature (**13 mK** nominal)	**15 mK** max operating temperature (*nominal to be measured*)
5% and **3.5%** precision level for *h* and *J*	**3.5%** and **2%** precision level for *h* and *J*	*To be measured*
20 us annealing time **12 ms** programming time	**5 us** annealing time (4X better) **12 ms** programming time	**5 us** annealing time **9 ms** programming time (25% better) **New:** anneal offset, pause, quench
6 graph connectivity per qubit	**6** graph connectivity per qubit	**6** graph connectivity per qubit

* Implements only an oracle that offers quality solutions to the Ising problem

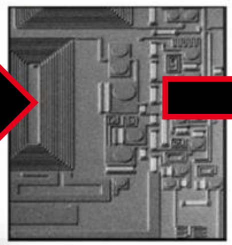
THE ANALOG QUANTUM COMPUTER ("HARDWARE")

Qbit implementation
-Rf SQUID* Flux Qbit
-Compound-Compound Josephson Junction

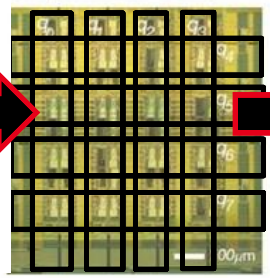




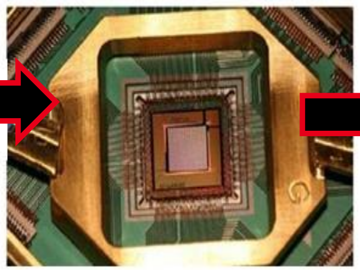
Niobium on silicon



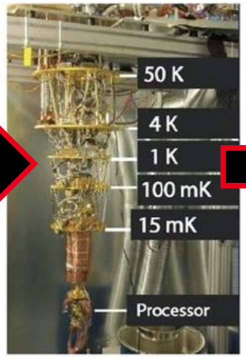
8-qbit unit cell



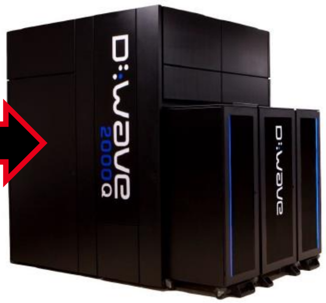
1152-qbit « Washington » chip

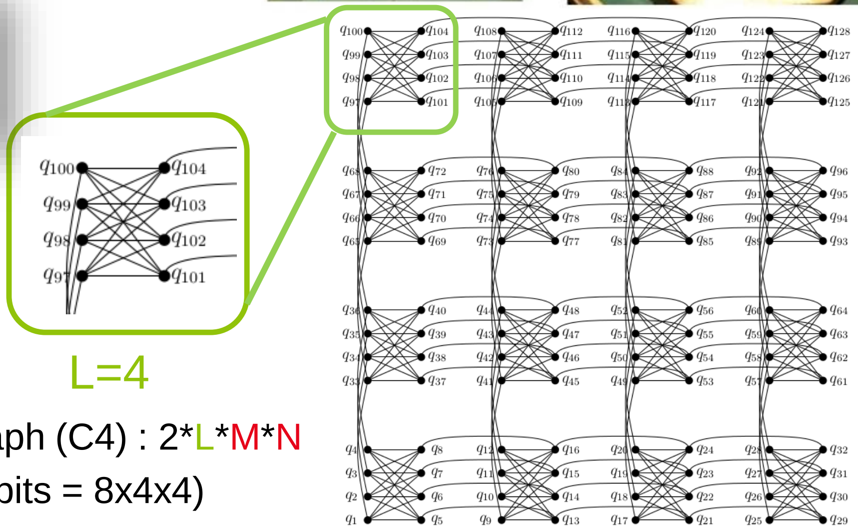


Pulse tube dilution refrigerator



Magnetically shielded enclosure (10^{-9} Tesla)





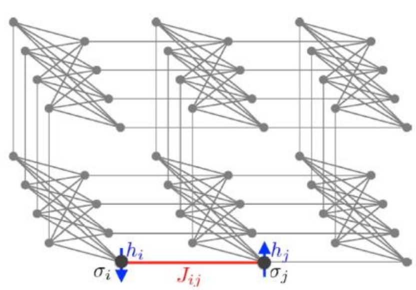
L=4

Chimera graph (C4) : 2*L*M*N qbits (128 qbits = 8x4x4)

Each qbit is connected according to a graph composed of 2 x 4 bipartite cells interconnected in a grid.

- Input: Instances of the Ising model -> map on a Chimera graph with weights h_i (qbits) and weights J_ij (connections).

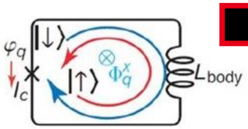
$$\mathcal{H}_{\text{Ising}} = \sum_{i \in V(G)} h_i \sigma_i^z + \sum_{ij \in E(G)} J_{ij} \sigma_i^z \sigma_j^z .$$

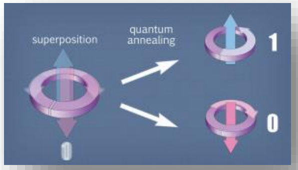


*Superconducting QUantum Interference Device

THE ANALOG QUANTUM COMPUTER ("HARDWARE")

Qbit implementation
-Rf SQUID* Flux Qbit
-Compound-Compound Josephson Junction





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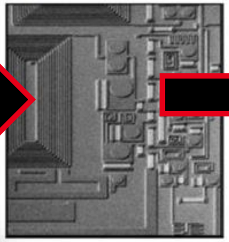
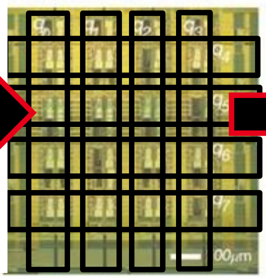
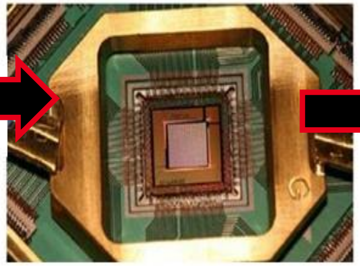


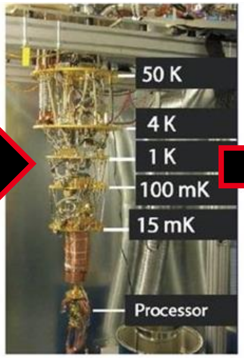
8-qbit unit cell



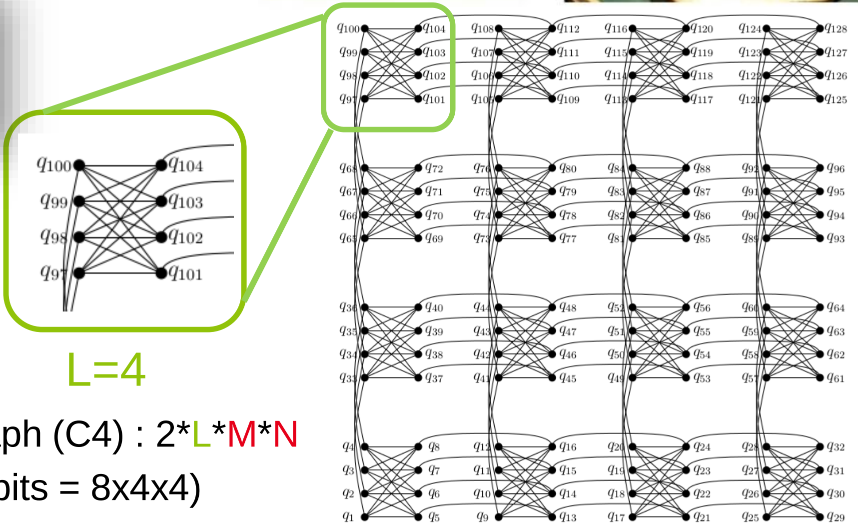
1152-qbit « Washington » chip



Pulse tube dilution refrigerator



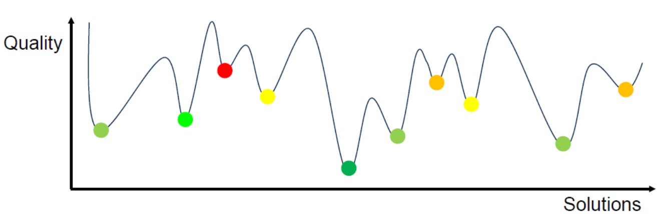
Magnetically shielded enclosure (10^{-9} Tesla)



Chimera graph (C4) : 2*L*M*N qbits (128 qbits = 8x4x4)

Each qbit is connected according to a graph composed of 2 x 4 bipartite cells interconnected in a grid.

- Ideally: Perform a quantum/adiabatic annealing to find one low energy state.
  -> Functionally similar to simulated annealing (not guaranteed to have an optimal solution).



Larger the problem size, greater number of qbits needed
-> Need for decomposition...
-> Interconnection topology constraints...

FROM QUBO TO ISING TO CHIMERA

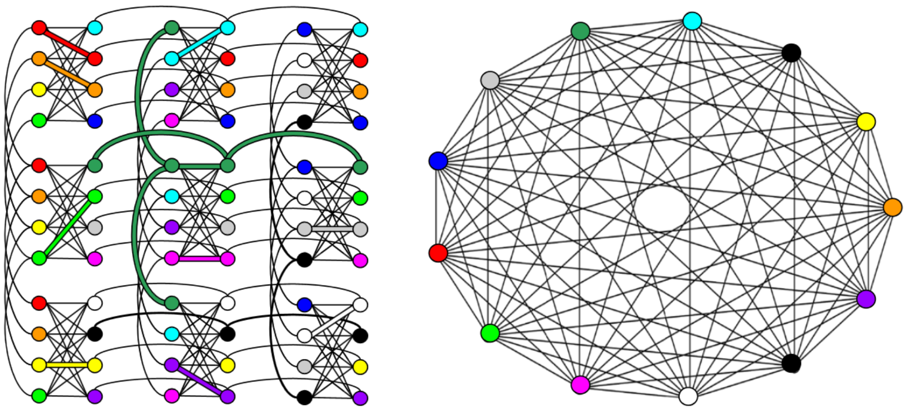
- Transformation of a QUBO* problem to an Ising Hamiltonian problem (NP-Hard):

$$\min_{X\in(0,1)^n} \sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij}.x_i.x_j \xrightarrow{\quad s_i = 2x_i - 1 \quad} \mathcal{H}_{\text{Ising}} = \sum_{i\in V(G)} h_i\sigma_i^z + \sum_{ij\in E(G)} J_{ij}\sigma_i^z\sigma_j^z.$$

- Mapping the QUBO matrix on the interacting physical qbits of the hardware graph:

- Approach: *1 variable → N qbits*

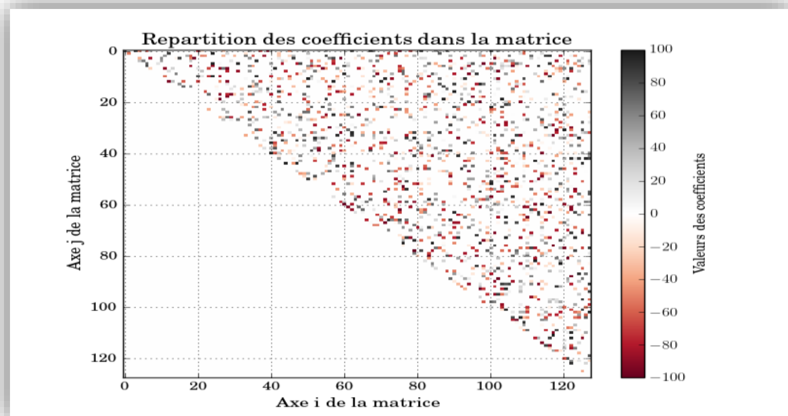
Not enough connections on qbits (hi) → max 6 edges !



Physical qbits are ONE variable!

- Approach: *1 variable → 1 qbit*

Not enough edges in the graph: 24x4x4 = 384 edges!



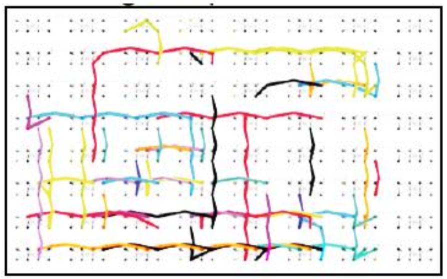
128 hi (qbits) + 7800 Jij (coupling terms)

*Quadratic Unconstrained Binary Optimization Problem

LIMIT OF A DECOMPOSITION APPROACH!?

Problem: very low qbit efficiency for dense matrices!

Up to 95% of physical qbits used for logical qbits connections



Very low physical qbit interconnection graph (quadratically)

- **Implemented solution in D-Wave tools:**

"*subdivide*" QUBO into resized sub-QUBOs to fit with the hardware architecture limitations.

- **Constraints:**
- Number of sub-problems: exp(prob_size/subprob_size)
- Risks of losing the initial problem structure

Requires several annealing invocations

→ Solving smaller problem sizes

- **Proposed approach**
- Exclude coefficients likely to have little impact on an optimal solution
- **Relaxation: try to "deepen" the matrix → A single call for annealing?**

OBJECTIFS

How to take advantage of the (low) qbits number?

Are there isomorphic relaxations within a Chimera graph which offers interesting solutions?

Is it easy to find such relaxations?

If this kind of relaxations exist → how complex is the corresponding algorithm?

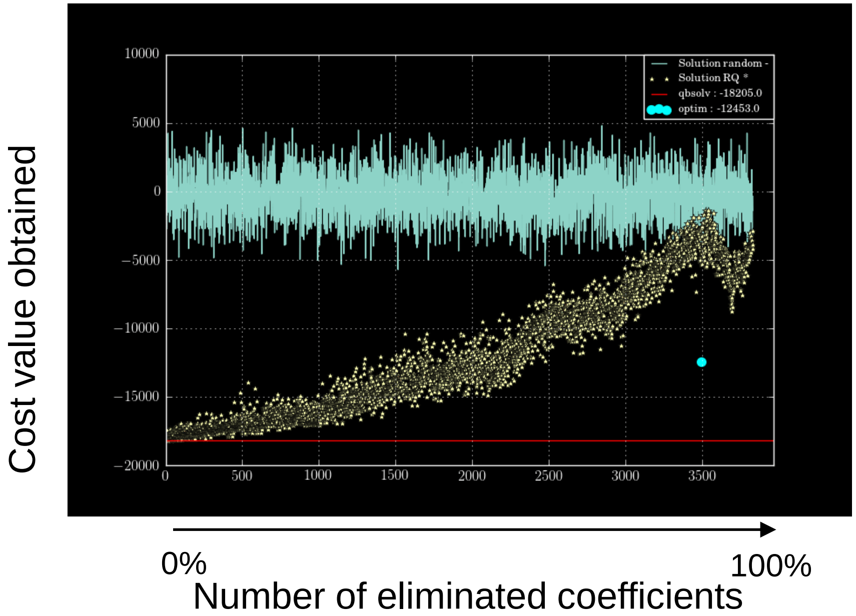
Else → hard to solve dense QUBO instances with a single call to the D-Wave quantum annealig

RESOLUTION APPROACH AND SEARCH ALGORITHM

- Relaxation $n_{coefficients}$QUBO = n_{edges}Chimera **<u>without interconnection topology</u>**

- First Step : Local search algorithm with a randomly selected subset $n_{coeffcieints}$QUBO
- Second Step : Randomly substituting a selected coefficient by zero
- Verification with annealing → Economic function computation of the problem

QUBO	Size	Edges	Density (%)	Qbsolv	$relax_1$	$rand$
factoring	127	703	10	-767	-538	13
bqp100-1	100	464	10	-12392	-11967	-548
bqp100-5	100	459	10	-9629	-9062	-557
Rand-1	128	509	8	-5912	-5158	-85
Rand-2	128	522	7	-5458	-4932	258
Rand-3	128	499	7	-6413	-6056	112
Rand-4	128	1496	18	-13159	-9901	-102
Rand-5	128	1570	19	-9940	-6524	23
Rand-6	128	1503	19	-12269	-9071	-570
Rand-7	128	3874	48	-16814	-8125	1175
Rand-8	128	3845	47	-18205	-11378	507
Rand-9	128	3819	47	-16443	-9609	-213
Rand-10	128	5769	71	-19175	-9174	429
Rand-11	128	5833	72	-21317	-12801	-555
Rand-12	128	5881	72	-22498	-12879	-2034
Rand-13	128	7866	97	-27762	-13984	-1057
Rand-14	128	7842	96	-23323	-11958	-2004
Rand-15	128	7839	96	-24758	-13276	-676



Competitive results with conventional annealing (except for high QUBO densities)
→ Experimentally the results are "good" quality solutions.

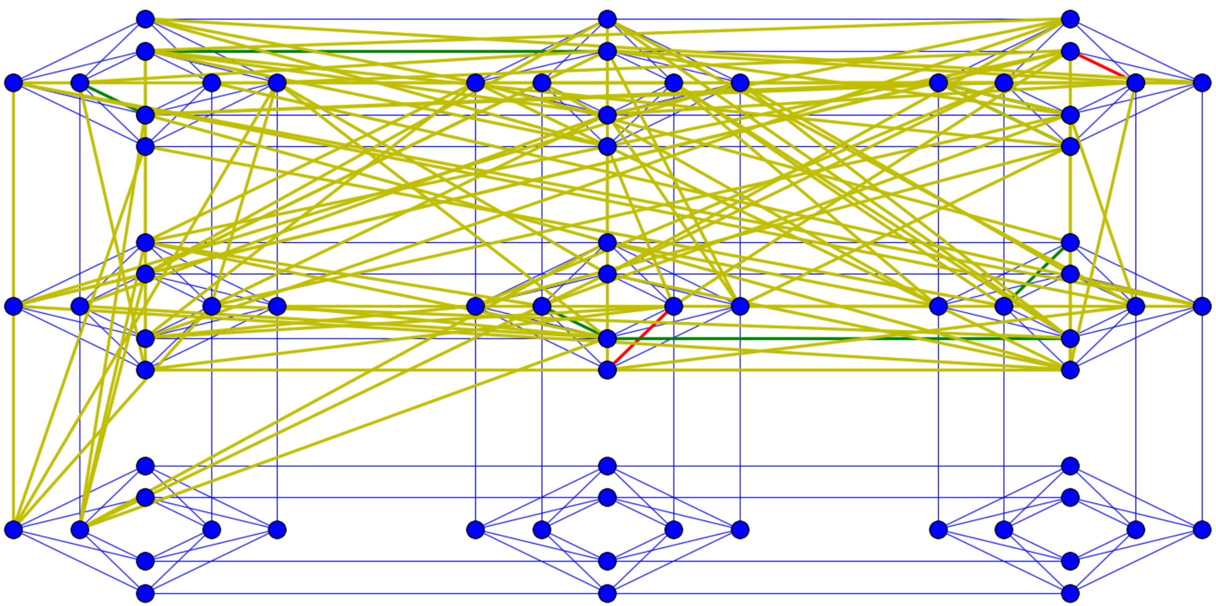
Generated matrices do not take into account topology!

RESOLUTION APPROACH AND TOPOLOGICAL CONSTRAINT

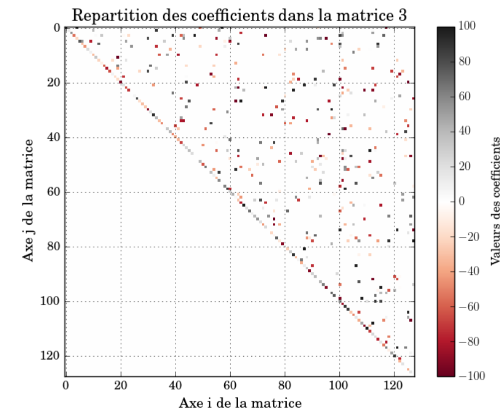
- Constraints related to the architecture of D-Wave → qbits interconnection topology represented in a quadratically less dense graph*.

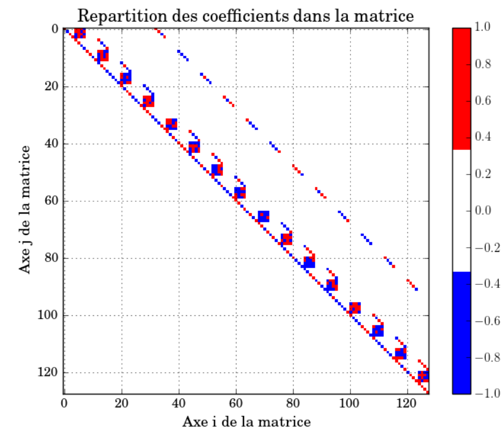
- When the QUBO problem graph is not isomorphic to this graph:

  Problem cannot be directly solved



Bqp100-1-> Total : 376/384 edges not mapped !!!

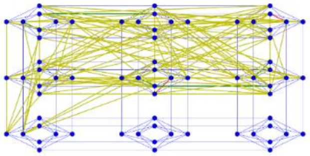




QUBO «fully» mapped on Chimera

* biparti graph grid 2x4.

RESOLUTION APPROACH AND TOPOLOGICAL CONSTRAINT



- Using local search

Solving optimization problems -> Finding the right solution in a set of candidate solutions: similar to annealing

<u>Goal*: Go through the set of "near" candidate solutions in order to find the better solution</u>

QUBO	Size	Edges	Density (%)	Qbsolv	$relax_1$	$relax_2$	$rand$
factoring	127	703	10	-767	-538	-202	13
bqp100-1	100	464	10	-12392	-11967	-5356	-548
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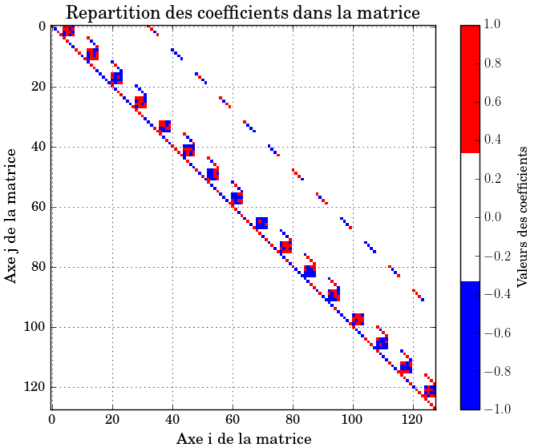


Mitigated results:

Low densities -> 30-40% selected -> up to 50% deviation.

High densities -> 85% selected -> Results close to the topology-free case.

*Vert, D., Sirdey, R., & Louise, S. (2019, April). On the limitations of the chimera graph topology in using analog quantum computers. In *Proceedings of the 16th ACM International Conference on Computing Frontiers* (pp. 226-229). ACM.

ANALYSIS AND PERSPECTIVES

By performing a large number of executions on matrices of different densities:

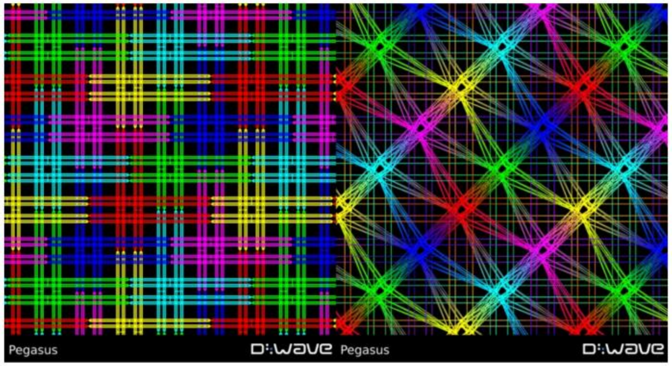
Possibility of obtaining solutions close to the pre-determined solutions.

Was it possible to easily obtain a graph which is isomorphic with Chimera and was it possible to have a solution close to the known optimal → Local search?

Two possibilities can be identified:

Work with other topologies? → Pegasus ?

Use quantum annealing on several isomorphic relaxations?



The new D-Wave internal architecture

Pegasus will have each qubit connected to 15 other qubits instead of 6 -> 2.5 times more connectivity

THANK YOU FOR YOUR ATTENTION

Centre de Saclay
Nano-Innov PC 172 - 91191 Gif sur Yvette Cedex

INSTITUT CARNOT CEA LIST

THE ANALOG QUANTUM COMPUTER ("ACCELERATOR?")

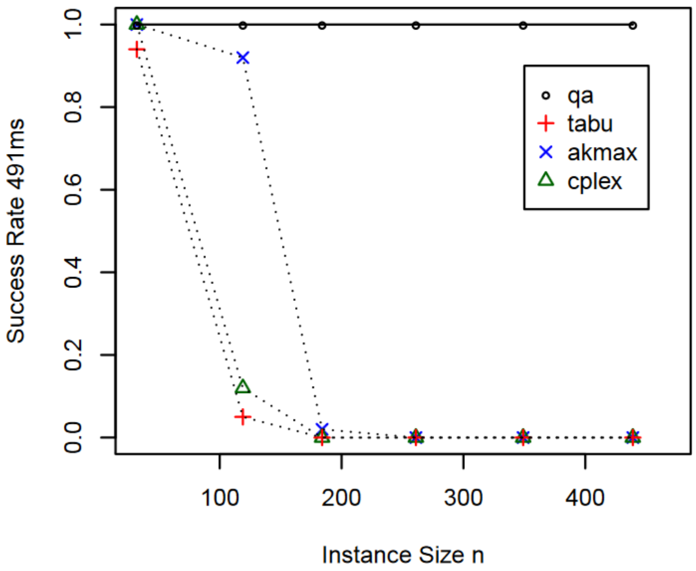
- Minimize the Ising model → (perhaps) faster than conventional annealing?
  → Hardware acceleration?

Performance comparisons among 4 solvers on ½ s
Isomorphic instance with Chimera*

<u>*Best solution cost admitted for each instance*</u>

Solver success rate → 3%.

Quantum Algo. success rate → 100% regardless of the problem sizes.



Proportion of best solutions found in 491ms of CPU time (tabu, akmax, cplex) and Quantum Algo. (hardware) McGeoch 2013

Results: Quantum Algo. remains competitive compare to a CPLEX solver running for up to 30 minutes!

Quantum Algo. appears to be equivalent to simulated annealing but with a quantum acceleration as a bonus?

*By the way, we ignore the problem's complexity of the studied case.

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